

## MATHCAD ACTIVITIES ON INFINITE SERIES AND TAYLOR POLYNOMIALS

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### ***Introduction***

Traditionally, calculus students at Old Dominion University find the material on infinite series and Taylor polynomials very challenging. This is largely due to the relatively high level of sophistication of the theory required; for example, students frequently struggle with the "maze" of convergence tests before they can successfully diagnose convergence or divergence of a series. Actually, it isn't just the theory, but the *concepts and ideas themselves* that are quite sophisticated here.

In 1992 the author, along with two colleagues, began developing a computer-based calculus course sequence, under a grant from the State Council for Higher Education in Virginia [1,2]. To enhance our students' learning of the infinite series material, a computer laboratory activity devoted to the subject was created.

In the Summer 1994, the author developed computer activities intended to provide an intuitive interpretation to some of the fundamental notions involved in studying infinite series and Taylor polynomials. These activities are described in the following sections.

### ***The Infinite Series Activity***

This activity is based on the Mathcad document presented in Figure 1. First, the  $n$ -th term expression,  $a_n$ , is defined; then the partial sums  $S_n$  are constructed recursively. Terms and partial sums are first plotted as "arrows", and then tabulated.

When Mathcad is in "automatic mode", changes made by a user go into effect immediately, affecting the rest of the document. By typing a different expression for  $a_n$ , different categories of series can then be illustrated:  $p$ -series, geometric series, alternating series etc.. The values assigned to the range variables  $n, i$  and  $m$  can also be

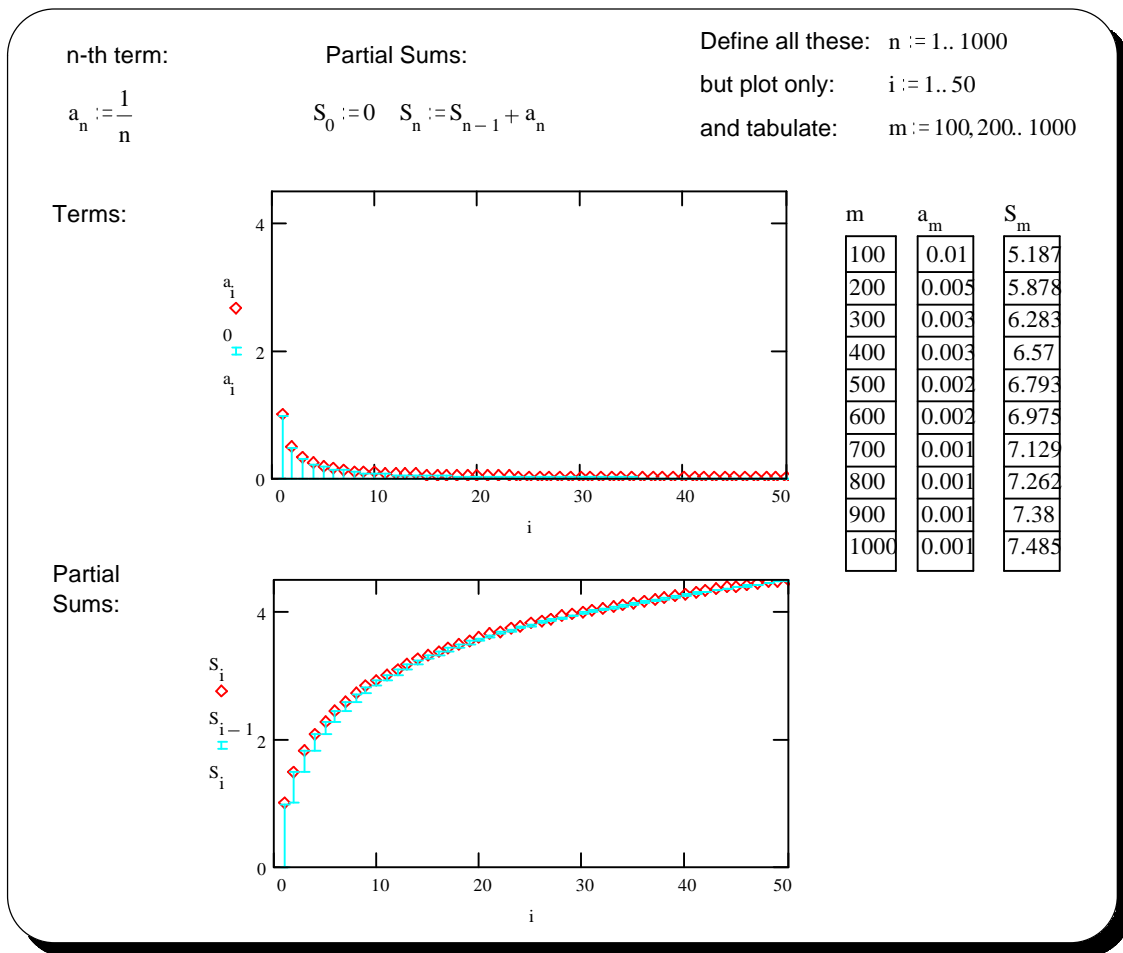


Fig. 1. The Mathcad screen corresponding to the Infinite Series activity.

modified to control which terms and partial sums are calculated, plotted and tabulated, respectively (subject to the restriction that the sets of values taken by  $i$  and  $m$  both be subsets of the set of values of  $n$ .) The instructor can perform such modifications to demonstrate the behavior of different series, and encourage students to experiment with different series.

The "arrow" plots of  $a_n$  and  $S_n$  are designed to show students how the terms of the series "build up", forming partial sums. For alternating series, these plots will aid a student's understanding of cancellations that take place when the partial sums are formed. Since students tend to have difficulty in distinguishing between the sequence of terms, and that of partial sums, we find illustrations that clearly contrast the two concepts (while emphasizing the relationship between them) very useful.

Another crucial distinction that has to be stressed is that between the *convergence* of the sequence of *terms*, and *convergence* of the *series*. The arrow plots and tables can be very helpful to provide a valuable visual and numerical illustration. In particular,

the professor can demonstrate divergence of a series whose sequence of terms converges to zero (which tends to be hard to grasp for many students).

### The Taylor Polynomials Activity

The objective of this activity is to let students "discover" Taylor polynomials and investigate their relationship with the function they approximate. On one hand, it serves as an informal introduction to the subject; on the other hand it guides students through a formal derivation of the Taylor formula. While what is described here could be carried out by the professor as a demonstration, it will be more effective if students can actively participate in the activity.

Upon loading the Mathcad document (see Fig. 2) the instructor should briefly explain the items present on the screen: the function definition, its plot, and a "table" of  $f(0)$  and the first four derivatives of  $f$  at 0. Also it should be noted that  $P(x) = a_0$ , a polynomial of zeroth degree has been defined, and its values, and those of its first four derivatives, at  $x=0$  are tabulated alongside those of  $f(x)$ . The students should be asked a question: *For what value of the constant  $a_0$  will the polynomial  $P(x) = a_0$  "mimic" the behavior of  $f(x)$  close to the value  $x = 0$ ?* Students are likely to produce the answer:  $a_0 = f(0) \approx 6.306$ . This is the best we can do with a polynomial of degree 0. We now raise the degree of the polynomial to 1 setting the new coefficient  $a_1$

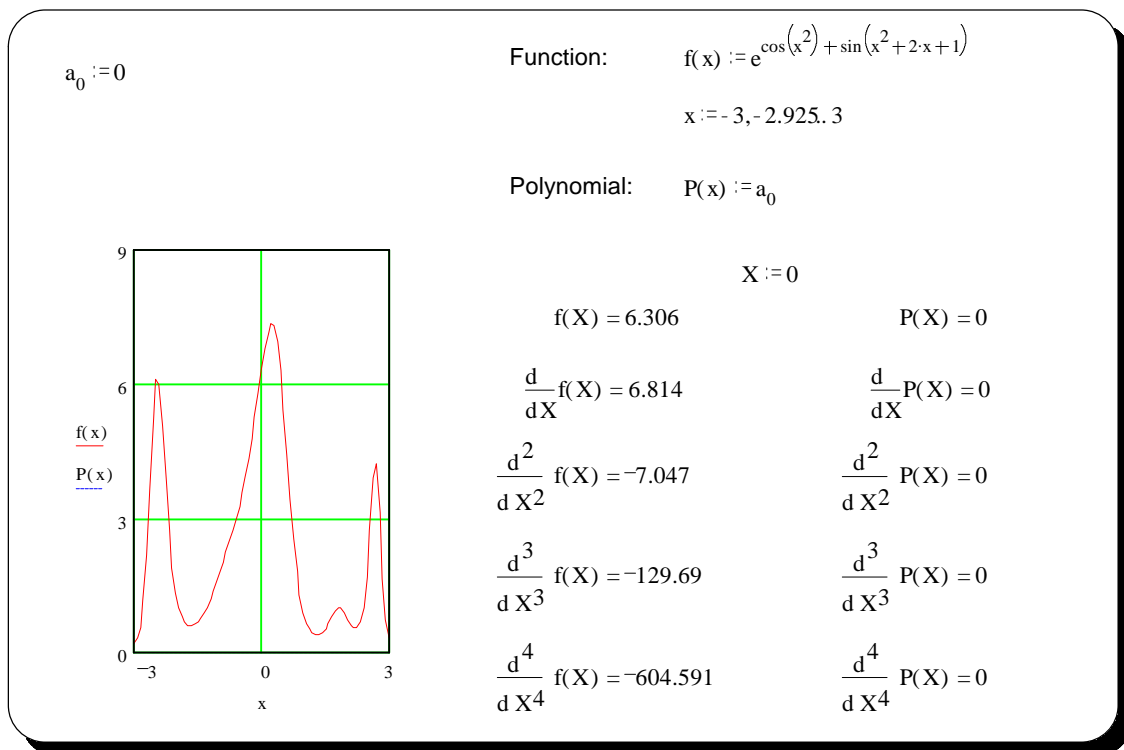


Fig. 2. The initial screen from the Taylor Polynomials activity.

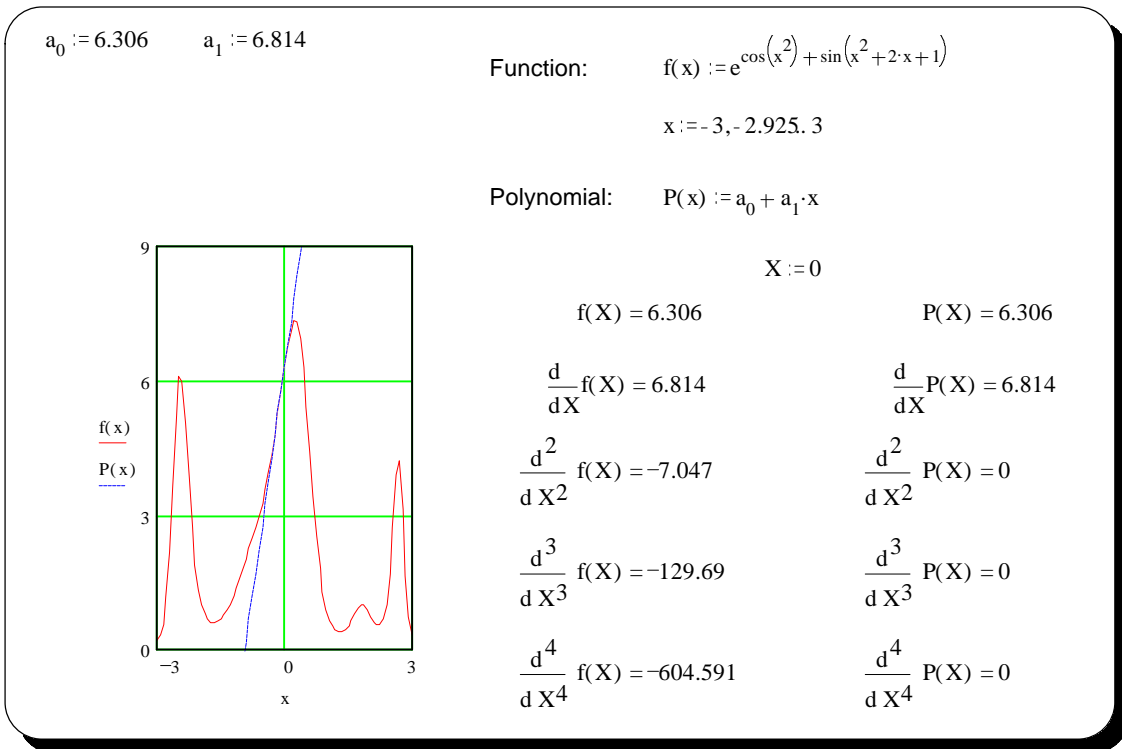


Fig. 3. An intermediate screen from the Taylor Polynomials activity.

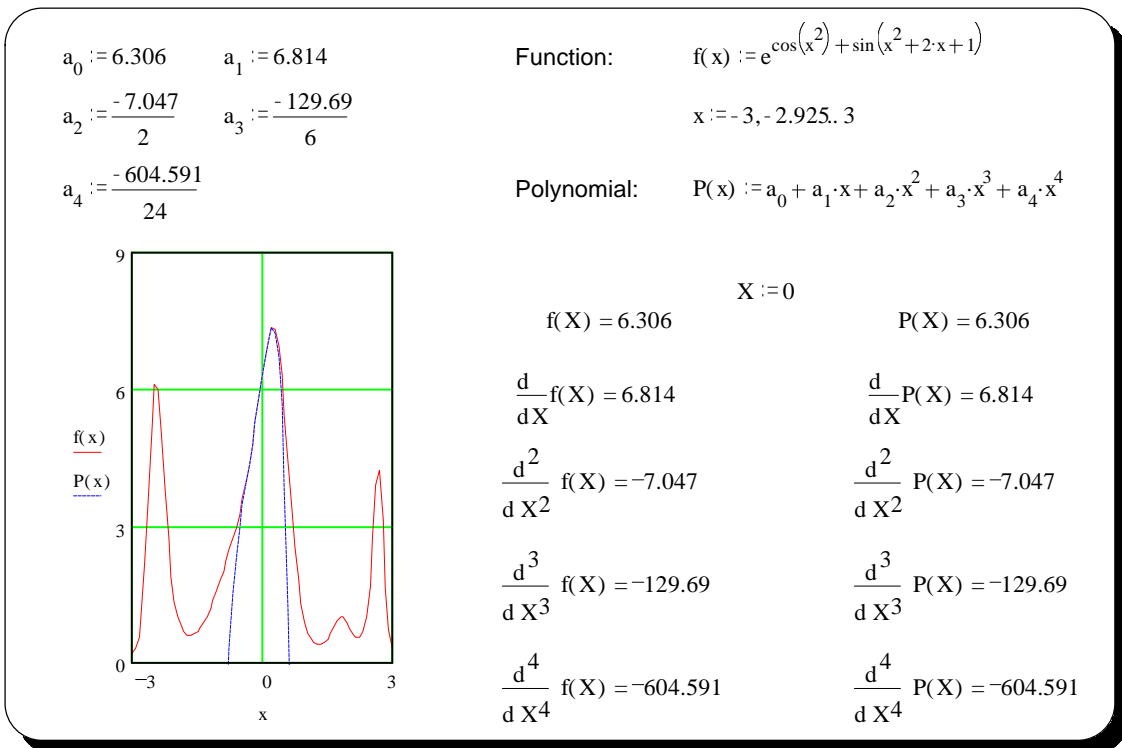


Fig. 4. The final screen from the Taylor Polynomials activity.

initially to **zero**. Question: *How should we use this extra freedom to do a better job following  $f(x)$  around  $x=0$  with our  $P(x)$ ?* After some experimentation, students will answer that  $a_1 = f'(0) \approx 6.814$ . We then remark that we now have a match in the first two rows of the derivative table (see Fig. 3.)

We can get a match in more rows if we raise the degree again. Set  $a_2 = 0$  and add the  $a_2x^2$  term to the expression defining  $P(x)$ . Again, let students experiment with the value of  $a_2$ . They will quickly discover that  $P''(0) = 2a_2$ . The instructor may want to take a break from the activity, and on the board show that for any polynomial

$$P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots + a_mx^m$$

we have  $P^{(n)}(0) = n! a_n$ . The activity will be concluded by raising the degree twice more, and setting the appropriate coefficient values. The final screen is presented in Fig. 4.

We believe this activity very well exemplifies Mathcad's potential in the classroom. Mathcad has a unique ability to arrange objects like assignments, expression evaluations and graphs in a logical (rather than interface-enforced) fashion. It was this feature, combined with Mathcad's dynamic response to user's input, that allowed this activity to occupy just one screen (without scrolling), while revealing a lot about the nature of Taylor polynomial approximation (in both numerical and visual fashion).

### ***Bibliography***

1. P. Bogacki, G. Melrose and P. R. Wohl, "The Old Dominion University Calculus Project" , Proceedings of the Sixth International Conference on Technology in Collegiate Mathematics, Addison-Wesley, Reading, MA, 1993.
2. P. Bogacki, G. Melrose and P. R. Wohl, "Mathcad 5.0 in Univariate Calculus" , Proceedings of the Seventh International Conference on Technology in Collegiate Mathematics, Addison-Wesley, Reading, MA, 1994.